ASYMPTOTIC KINETICS OF THERMAL BREAKDOWN OF SOLID DIELECTRICS

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The thermal breakdown of solid dielectrics includes, in a simplified form, three stages: 1) heating of the solid phase up to the temperature of the effective phase transition T_*; 2) isothermal transition into a conducting gaseous phase; and 3) flow of the latter phase. Assuming the electric potential φ of the conducting phase is constant, we can assume that the energy inflow owing to Joule dissipation occurs only in the first two stages. Assuming that the breakdown times are short, transport processes can be neglected. For simplicity we neglect free charges. We write Ohm's law in its simplest form:

$$\mathbf{j} = \sigma \mathbf{E}, \tag{1}$$

where $\mathbf{j}$ is the electric current density; $\mathbf{E}$ is the intensity of the electric field; and, σ is the conductivity. Confining our attention to the one-dimensional case, we shall take into account the spatial asymmetry of breakdown on the basis of a spherical description. The foregoing simplifications of the problem were adopted in order to simplify the analysis. The chief simplification is neglecting the transport processes. It is in this sense that the present description is asymptotic. The remaining simplifications are not fundamental and can be removed if necessary. At the end of this paper, in particular, a possible method for taking into account the displacement current is briefly discussed.

Thus the problem reduces to a system consisting of the energy equations and the particular Maxwell's equations, describing the first two stages:

$$\frac{\partial}{\partial t}(\gamma u) = (\mathbf{jE}) \equiv jE; \tag{2}$$

$$\operatorname{div} \mathbf{j} \equiv \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 j) = 0; \tag{3}$$

$$\operatorname{div} \mathbf{D} \equiv \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D) = 0. \tag{4}$$

Here γ is the density; u is the internal energy; $\mathbf{D} = \varepsilon \mathbf{E}$ is the electric induction (ε is the dielectric constant); t is the time; and r is the radius ($R_1 \leqslant r \leqslant R_2$). When the total current I(t) is given, Eq. (3) is employed; when the external voltage U(t) is given, (4) is employed. At the boundaries of the phases either j [according to (3)] or D [according to (4)] is continuous. In addition, as usual, it is assumed that T and φ are continuous.

We shall first study the simplest situation when $I(t)$ is given. The general solution of (2), according to the integral of (3)

$$2\pi r^2 j = I(t), \tag{5}$$

has the form

$$\int_{T_0}^{T} \sigma d(\gamma u) = \frac{1}{(2\pi)^2}\int_0^t I^2(t')\,dt'\,\frac{1}{r^4} + f(r), \tag{6}$$

where the arbitrary function f(r) = 0, according to the initial condition $T(r, 0) = T_0$. At the first stage $u = c(T - T_0)$ (c is the heat capacity), and assuming for simplicity that γ, c = const, we obtain

$$\gamma c \int_{T_0}^{T} \sigma(T')\,dT' = \frac{1}{(2\pi)^2}\int_0^t I^2(t')\,dt'\,\frac{1}{r^4}, \tag{7}$$

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so that the moment t_0 at which $T(R_1, t)$ reaches the value T_* is determined from (7) with $r = R_1$: $\gamma c \int_{T_0}^{T_*} \sigma(T')\,dT' = \frac{1}{(2\pi)^2} \int_0^{t_0} I^2(t')\,dt' \frac{1}{R_1^4}$.

At times $t > t_0$ a front corresponding to the start of the phase transition, whose coordinate $\rho_1(t)$ is determined from (7) with $r = \rho_1$,

$$\gamma c \int_{T_0}^{T_*} \sigma(T')\,dT' = \frac{1}{(2\pi)^2} \int_0^{t} I^2(t')\,dt' \frac{1}{\rho_1^4} \tag{8}$$

will pass into the dielectric. For $R_1 \leqslant r \leqslant \rho_1$ a phase transition occurs, and the degree of completion of the transition $\xi(r, t)$ $(0 \leqslant \xi \leqslant 1)$ is obtained from the equation

$$\gamma \left[c \int_{T_0}^{T_*} \sigma(T')\,dT' + \sigma(T_*)\lambda\xi \right] = \frac{1}{(2\pi)^2} \int_0^{t} I^2(t')\,dt' \frac{1}{r^4} \tag{9}$$

[λ is the heat of the phase transition ($u = c(T_* - T_0) + \lambda\xi$)]. The time t_1 of completion of the phase transition ($\xi = 1$) at $r = R_1$ is determined from (9):

$$\gamma \left[c \int_{T_0}^{T_*} \sigma(T')\,dT' + \sigma(T_*)\lambda \right] = \frac{1}{(2\pi)^2} \int_0^{t_1} I^2(t')\,dt' \frac{1}{R_1^4}.$$

For $t > t_1$ the front corresponding to the end of the phase transition, whose coordinate $\rho_2(t)$ is determined from (9) with $r = \rho_2$

$$\gamma \left[c \int_{T_0}^{T_*} \sigma(T')\,dT' + \sigma(T_*)\lambda \right] = \frac{1}{(2\pi)^2} \int_0^{t} I^2(t')\,dt' \frac{1}{\rho_2^4} \tag{10}$$

will pass into the dielectric. Inside the region of the phase transition ($\rho_2 \leqslant r \leqslant \rho_1$) we find $\xi(r, t)$ as before from (9). The time t_2 at which the start of the phase transition reaches the outer surface of the dielectric is determined from (7) with $r = R_2$: $\gamma c \int_{T_0}^{T_*} \sigma(T')\,dT' = \frac{1}{(2\pi)^2} \int_0^{t_2} \times$ $I^2(t')\,dt' \frac{1}{R_2^4}$. Finally, the time t_3 at which the end of the phase transition reaches the outer surface of the dielectric – the total breakdown time – is obtained from (9) with $r = R_2$: $\gamma \left[c \int_{T_0}^{T_*} \sigma(T')\,dT' + \sigma(T_*)\lambda \right] = \frac{1}{(2\pi)^2} \int_0^{t_3} I^2(t')\,dt' \frac{1}{R_2^4}$. It is easy to see that the inequality $t_0 < t_{1,2} < t_3$ holds.

The relation between t_1 and t_2 is determined by the quantities $\int_{T_0}^{T_*} \sigma(T')dT'/[\sigma(T_*)\lambda]$ and R_2/R_1. Figure 1 shows for convenience the r–t diagram with the curves $\rho_1(t)$ and $\rho_2(t)$ for I = const, when $t_1 - t_0 < t_3 - t_2$, and $\int_{T_0}^{T_*} \sigma(T')\,dT'/(\sigma_*\lambda) < 1$, when $t_0 < t_1 - t_0$. The numbers 1-3 refer to the conducting phase, the region of the phase transition, and the dielectric.

We return now to the situation when U(t) is given. For simplicity we assume that ε = const. The integrals (4) have the form

$$E = g_1(t)/\varepsilon r^2, \quad \varphi = g_1(t)/\varepsilon r + g_2(t) \tag{11}$$

($g_{1,2}$ are arbitrary functions). At the stage 1

$$E = \frac{U}{1/R_1 - 1/R_2} \frac{1}{r^2} \tag{12}$$

and the integral (2)

$$\gamma c \int_{T_0}^{T} \frac{dT'}{\sigma(T')} = \frac{1}{(1/R_1 - 1/R_2)^2} \int_0^{t} U^2(t')\,dt' \frac{1}{r^4} + f(r). \tag{13}$$

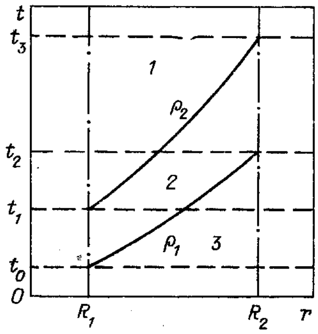

Fig. 1

Here the arbitrary function f(r) = 0 because of the initial condition. The moment t_0 at which $T(R_1, t)$ reaches the value T_* is determined from (13) with $r = R_1$:

$$\gamma c\int_{T_0}^{T_*}\frac{dT'}{\sigma(T')}=\frac{1}{(1/R_1-1/R_2)^2}\int_0^{t_0}U^2(t')\,dt'\,\frac{1}{R_1^4}.$$

For $t > t_0$ the coordinate corresponding to the start of the phase transition $\rho_1(t)$ is found from (13) with $r = \rho_1$:

$$\gamma c\int_{T_0}^{T_*}\frac{dT'}{\sigma(T')}=\frac{1}{(1/R_1-1/R_2)^2}\int_0^{t}U^2(t')\,dt'\,\frac{1}{\rho_1^4}. \tag{14}$$

For $R_1 \leqslant r \leqslant \rho_1$ a phase transition occurs; the degree of completion $\xi(r, t)$ is obtained from the equation

$$\gamma\left[c\int_{T_0}^{T_*}\frac{dT'}{\sigma(T')}+\frac{\lambda}{\sigma(T_*)}\xi\right]=\frac{1}{(1/R_1-1/R_2)^2}\int_0^{t}U^2(t')\,dt'\,\frac{1}{r^4}. \tag{15}$$

The moment t_1 at which the phase transition stops ($\xi = 1$) is determined from (15) with $r = R_1$: $\gamma\left[c\int_{T_0}^{T_*}\frac{dT'}{\sigma(T')}+\frac{\lambda}{\sigma(T_*)}\right]=\frac{1}{(1/R_1-1/R_2)^2}\int_0^{t_1}U^2(t')\,dt'\,\frac{1}{R_1^4}.$

For $t > t_1$ a front corresponding to the end of the phase transition, whose coordinate $\rho_2(t)$ is found as follows, passes into the dielectric. We write the integrals (4) in the form

$$E=\frac{1}{\varepsilon r^2}\sqrt{\frac{dh_1(t)}{dt}},\quad \varphi=\frac{1}{\varepsilon r}\sqrt{\frac{dh_1(t)}{dt}}+h_2(t) \tag{16}$$

($h_{1,2}$ are arbitrary functions). Because the potential of the conducting phase is constant U(t) is applied to the gap $\rho_2(t) - R_2$, i.e.,

$$U=\frac{1}{\varepsilon}\left(\frac{1}{\rho_2}-\frac{1}{R_2}\right)\sqrt{\frac{dh_1}{dt}}. \tag{17}$$

The integral (2) in the stage 1 is

$$\gamma c\int_{T_0}^{T}\frac{dT'}{\sigma(T')}=\frac{h_1}{\varepsilon^2}\frac{1}{r^4}+f(r), \tag{18}$$

where the arbitrary function f(r) is calculated from the condition that (18) be identical to (13) at $t = t_1$, so that

$$h_1(t)=h_1(t_1)+\varepsilon^2\left[\gamma c\int_{T_0}^{T}\frac{dT'}{\sigma(T')}r^4-\frac{1}{(1/R_1-1/R_2)^2}\int_0^{t_1}U^2(t')\,dt'\right]. \tag{19}$$

From (17) and (19) with $r = \rho_1$ we obtain

$$4\left(\frac{1}{\rho_2}-\frac{1}{R_2}\right)^2\rho_1^3\frac{d\rho_1}{dt}=\frac{U^2}{\gamma c\int\limits_{T_0}^{T_*} dT'/\sigma(T')}. \quad (20)$$

The integral (2) at the stage 2

$$\gamma\left[c\int\limits_{T_0}^{T_*}\frac{dT'}{\sigma(T')}+\frac{\lambda}{\sigma(T_*)}\xi\right]=\frac{h_1}{\varepsilon^2}\frac{1}{r^4}+f(r), \quad (21)$$

where the arbitrary function f(r) is determined from the condition that (21) be identical to (15) at $t = t_1$, so that

$$h_1(t)=h_1(t_1)+\varepsilon^2\left\{\gamma\left[c\int\limits_{T_0}^{T_*}\frac{dT'}{\sigma(T')}+\frac{\lambda}{\sigma(T_*)}\xi\right]r^4-\frac{1}{(1/R_1-1/R_2)^2}\int\limits_0^{t_1}U^2(t')dt'\right\}. \quad (22)$$

From (17) and (22) with $r = \rho_2$ we find

$$4\left(\frac{1}{\rho_2}-\frac{1}{R_2}\right)^2\rho_2^3\frac{d\rho_2}{dt}=\frac{U^2}{\gamma\left[c\int\limits_{T_0}^{T_*} dT'/\sigma(T')+\lambda/\sigma(T_*)\right]}. \quad (23)$$

From the system (20) and (23) we obtain the final integrals

$$\frac{\rho_2^4-\rho_2^4(t_1)}{\rho_1^4-\rho_1^4(t_1)}=\frac{c\int\limits_{T_0}^{T_*} dT'/\sigma(T')}{c\int\limits_{T_0}^{T_*} dT'/\sigma(T')+\lambda/\sigma(T_*)}<1; \quad (24)$$

$$(1/3)\,R_2^2(6x^2-8x^3+3x^4)\Big|_{x_1}^{x}=\frac{\int\limits_{t_1}^{t}U^2(t')\,dt'}{\gamma\left[c\int\limits_{T_0}^{T_*} dT'/\sigma(T')+\lambda/\sigma(T_*)\right]} \quad (25)$$

[$(\rho_2(t_1)=R_1$, $x=\rho_2/R_2\leqslant 1$, and $x_1 = R_1/R_2 < 1$]. The time t_3 of breakdown of the dielectric is determined from (25) with $\rho_2 = R_2$ ($x = 1$):

$$\frac{1}{3}\frac{1}{R_2^2}(R_2-R_1)^3(R_2+3R_1)=\frac{\int\limits_{t_1}^{t_3}U^2(t')\,dt'}{\gamma\left[c\int\limits_{T_0}^{T_*} dT'/\sigma(T')+\lambda/\sigma(T_*)\right]}.$$

Finally, we have the convenient expression

$$\gamma\left[c\int\limits_{T_0}^{T_*}\frac{dT'}{\sigma(T')}+\frac{\lambda}{\sigma(T_*)}\right]=\frac{3}{1+2x_1}\int\limits_0^{t_3}\left[\frac{U(t')}{R_2-R_1}\right]^2 dt', \quad (26)$$

which in the limit $x \to 1$ transforms into the obvious result for the two-dimensional case, when the phase transition passes uniformly through the entire volume of the dielectric. Because of the inequality $3/(1+2x_1)>1$ it follows from (26) that the spatial asymmetry of the breakdown shortens its duration.

Taking into account free charges leads to the appearance of a displacement current, which is physically equivalent to relaxation of the electric field strength to its equilibrium value. For a solid dielectric, because of the redistribution of the internal energy, the general equation for the energy neglecting transport processes in the one-dimensional spherical case has the form (analogous to [1])

$$\frac{\partial}{\partial t}\left[\gamma u+\left(\frac{\partial T\varepsilon}{\partial T}\right)_\gamma\frac{E^2}{8\pi}\right]+\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(j+\frac{1}{4\pi}\frac{\partial D}{\partial t}\right)\varphi\right]=0 \quad (27)$$

or

$$\frac{\partial}{\partial t}(\gamma u) = jE + \left\{\frac{\partial \varepsilon}{\partial t}\frac{E^2}{8\pi} - \frac{\partial}{\partial t}\left[T\left(\frac{\partial \varepsilon}{\partial T}\right)_\gamma \frac{E^2}{8\pi}\right]\right\}, \tag{28}$$

which reduces to (2) with ε = const. The equation of continuity of the total current [1] is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(j + \frac{1}{4\pi}\frac{\partial D}{\partial t}\right)\right] = 0, \tag{29}$$

whose first integral is

$$2\pi r^2\left(j + \frac{1}{4\pi}\frac{\partial D}{\partial t}\right) = I(t). \tag{30}$$

The system (28) and (30) is closed by Ohm's law (1) and the condition of potentiality $E = -\partial\varphi/\partial r$. Integration of this system is complicated by the fact that ε and σ depend on T (for a solid it may be assumed that γ = const). Equation (27) differs from the analogous relation in [1] by a redefinition of the internal energy and Poynting's vector.

LITERATURE CITED

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MEASUREMENT OF THE TOTAL SCATTERING CROSS SECTIONS OF INERT GASES IN THE RELATIVE ENERGY RANGE 7-17 eV

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The description of collisional processes in rarefied gases requires information about the interaction potentials. Theoretical and experimental data have now been accumulated on the short-range intermolecular forces. The status of this field is reviewed in [1-4]. The chief results were obtained primarily from experiments on scattering of high-energy beams (E ~ 1 keV) by small angles ($\theta \sim 10^{-2}$ rad) or from measurements of the attenuation of beams passing through a layer of scattering gas (gas target). Empirical values of the parameters in power-law and exponential potentials for different atomic and molecular gases are collected together in [1].

The region of moderate interaction energies (~10 eV) has been less studied, since the question of producing monoenergetic neutral particle beams in the region 1-10 eV has yet to be resolved. In the last few years, problems for which reliable information about the interaction potentials in this energy range is required in order to obtain qualitative and quantitative results have appeared. They include the questions of the formation of the characteristic external atmosphere (CEA) around aircraft at high altitudes. One of the chief mechanisms for transport of pollutants to the sensitive elements of the environment in the formation of CEA are return flows, determined by collisions between the pollutant particles and the particles of the incident flow.

In this case the problem reduces to summing the particle fluxes on the corresponding surface element dS of the body in the flow [5] $dN = n_1 n_2 d\sigma g_{21} d\tau dS$, etc., where n_1 and n_2 are the particle densities in the incident flow and the products of mass release from the structural surfaces (as a result of desorption, degassing, sublimation, evaporation, etc.) in the volume element of the physical space dτ, $g_{21} = |\mathbf{v}_2 - \mathbf{v}_1|$ is the relative velocity of the colliding particles (the index 1 refers to the particles in the incident flow and the index 2 refers to the mass-release particles); dσ is the differential cross section for scattering into the solid angle dω, at which the element dS can be seen from the center of the volume dτ.

In the case of elastic-sphere molecules the differential scattering cross section in the coordinate system fixed to dS can be represented in the form

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